S-Duality and Exact Type IIB Superstring Backgrounds

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Abstract

A geometrical approach in the non-symmetric connection framework is employed to examine the issue of higher order α' corrections to D=10 type IIB superstring backgrounds with a covariantly constant null Killing isometry and non-zero Ramond-Ramond field content. These describe generalized supersymmetric string waves and were obtained recently by us through the S-duality transformations of purely NS-NS plane wave backgrounds. We find that the backgrounds are exact subject to the existence of certain field redefinitions and provided certain restrictive conditions are satisfied.

1. Introduction

The past two years have witnessed remarkable developments in the understanding of non-perturbative aspects of superstring theories. Although a clear conception of the dynamical issues in the non-perturbative regime is yet to emerge, symmetry considerations have yielded an exciting glimpse. The central role in these investigations have been played by the *duality symmetries* of superstring theories, both the perturbative T-dualities [1] and the non-perturbative S [2, 3, 4] and U [5] dualities. The latter relates weak and strong coupling regimes and electricaly charged perturbative string states with magneticaly charged solitons. They arise as global non-compact symmetries in the low energy effective supergravity theories which are conjectured to extend to the full quantum superstring theory as a discrete version, being broken by instanton effects. As an example we have the global $SL(2,R)$ symmetry of the type IIB supergravity in ten dimensions [5, 6] extending to the full type IIB superstring theory as $SL(2,Z)$ in ten dimensions, and relating the NS-NS sector to the R-R sector. This leads to the possibility of generating backgrounds with R-R field contents starting from purely NS-NS configurations. We will refer to both the classical continuous symmetry and the discrete symmetry of the quantum string theory as S-duality.

The R-R field content of the type II superstrings were not amenable to a sigma model analysis [7] as they lacked a conformal field theory description on the world sheet [8]. Very recently the conformal field theory for the superstring backgrounds which are charged under the R-R gauge fields have been identified to be that of open superstrings with Dirichlet boundary conditions [9]. However such an interpretation for backgrounds uncharged under the R-R sector for *eg.* supersymmetric plane waves is still obscure. The tree level equations of motion for these type II backgrounds, are of course those of the type II supergravity which are field theory limits of type II superstrings. In contrast, the higher order contributions to the tree level equations of motion are still intractable for these non-p-brane like backgrounds, owing to the lack of an explicit sigma model description. For recent results see refs. [10]. Certain classes of

string backgrounds are found to have all higher order contributions to the equations of motion to be identicaly zero [11, 12, 13, 14, 15] . Consequently the tree level equations are exact. These higher order arguments are based on sigma model considerations and are hence not applicable to the R-R sector of the non-brane like backgrounds considered here. However, there are a class of string backgrounds with a covariantly constant null Killing isometry [11, 13] for the bosonic and heterotic versions of which, there exists a purely geometrical approach to demonstrate that the higher order terms are vanishing [12, 16, 17, 18]. This analysis makes no reference to sigma models and are thus particularly suitable for the R-R sector of type IIB superstring backgrounds. The simplest example in this class are the plane waves and the general class is referred to as the K-models [13]. The K-model backgrounds posess vector like couplings on the world sheet which on compactification provides background gauge fields. The bosonic and heterotic K-models describe strings propagating in a uniform magnetic field [19] which may be formulated as an exact conformal field theory.

In an earlier article [20], we had demonstrated that starting from a simple plane wave background [16] with a purely NS-NS configuration it is possible to generate non trivial R-R field content using the $SL(2, R)$ S-duality symmetry of the type IIB supergravity equations of motion in ten dimensions. This results in type IIB plane wave backgrounds. Furthermore using the geometrical considerations referred earlier, it could be shown that these type IIB backgrounds were exact to all orders in α'. This analysis was subsequently generalized [21] to the case of the K-models [13, 15] (with a Brinkmann metric [22]) and a flat transverse part in ten dimensions. We showed using the same geometrical analysis that the type IIB backgrounds obtained through the S-Duality were exact, provided the vector like couplings in the metric were linear functions of the transverse coordinates [21]. For the case when the couplings are arbitrary functions, there are explicit higher order contributions to the equations of motion [15] and a straightforward application of the geometrical method was not viable. For the K-model backgrounds with purely NS-NS field content it was shown

by Duval *et. al.* [17] that the geometrical approach may be modified in the non-symmetric connection framework of Osborn [23]. For the one-loop beta functions, it was possible to show in this fashion that the higher order contributions from the metric and the antisymmetric tensors to the equations of motion cancel provided the vector like couplings enter the theory in a chiral fashion. They further argued that such cancellations will persist at all higher orders in α' provided certain field redefinitions exist in addition such that all higher order terms may be expressed in a generic fashion. This was later confirmed by an explicit higher order sigma model analysis [15].

In the present article we apply the non-symmetric connection framework to the type IIB K-model like superstring backgrounds obtained by us in [21]. For these plane wave like type IIB backgrounds there are no sigma model interpretations and the geometrical approach alluded to earlier seems to be the only way of considering higher order corrections. A modification to the approach in [17] is however required to accommodate the R-R sector. Using this methodology we investigate the issue of higher order contributions to the equations of motion for the case when the vector like couplings are arbitrary functions. We observe that subject to the existence of certain field redefinitions and the vector like couplings being chiral, the type IIB backgrounds [21] are also exact to all orders in α'. We examine the generic higher order contributions at two-loops explicitly. From this we provide a strong argument for the existence of such field redefinitions. We observe that for the backgrounds considered here, the $SL(2, R)$ transformations preserves the structure of α' corrections subject to the existence of appropriate field redefinitions.

The article is divided into five sections. In Section-2 we briefly review the non symmetric connection approach of Osborn and describe the application to K-model string backgrounds due to Duval *et. al.* modified to include the dilaton field. In Section-3 we present the solution generating technique and describe the type IIB background obtained by us in [21]. In Section-4 we express the background field

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dynamics using a modified version of the non symmetric connection which is consistent with the type IIB field content. We also address the issue of higher order corrections to the equations of motion through a purely geometrical analysis. A summary and discussion of our results are presented in the concluding Section-5.

2. Non-Symmetric Connections and K-Model String Backgrounds.

The non-symmetric connection approach [17, 23] is a method particularly suited for an unified description of the background field dynamics of both the metric and the antisymmetric tensor field describing a string background configuaration. We briefly review this here and also include the dilaton contribution. The approach proceeds as follows; if $G_{\mu\nu}, B_{\mu\nu}$ and ϕ describes a consistent string background then one defines,

$$\mathcal{E}_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} \tag{1}$$

and the non-symmetric connection is defined to be,

$$\Gamma^{+\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} + H^{\rho}{}_{\mu\nu} \tag{2}$$

which may be expressed in the following way as;

$$\Gamma^{+}_{\rho\mu\nu} = \frac{1}{2}[\partial_\nu \mathcal{E}_{\rho\mu} + \partial_\mu \mathcal{E}_{\nu\rho} - \partial_\rho \mathcal{E}_{\nu\mu}]. \tag{3}$$

Here $H_{\mu\nu\rho} = \frac{1}{2}(\partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu})$ is the field strength corresponding to the antisymmetric tensor $B_{\mu\nu}$. The generalized curvature corresponding to these connections in eqn.(3) are

$$R^{+\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{+\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{+\rho}{}_{\mu\sigma} + \Gamma^{+\rho}{}_{\mu\alpha}\Gamma^{+\alpha}{}_{\nu\sigma} - \Gamma^{+\rho}{}_{\nu\alpha}\Gamma^{+\alpha}{}_{\mu\sigma}. \tag{4}$$

With this the generalized curvature $R^{+}_{\lambda\mu\nu\kappa}$ is antisymmetric in the first and the second pair of indices. The generalized curvature may also be expressed in terms of the usual Christoffel connections and the torsion in the following way;

$$R^{+\rho}{}_{\sigma\mu\nu} = R^{\rho}{}_{\sigma\mu\nu} + D_\mu H^{\rho}{}_{\nu\sigma} - D_\nu H^{\rho}{}_{\mu\sigma} + H^{\rho}{}_{\mu\alpha}H^{\alpha}{}_{\nu\sigma} - H^{\rho}{}_{\nu\alpha}H^{\alpha}{}_{\mu\sigma}. \tag{5}$$

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It is possible to derive a generalized Bianchi identity,

$$D^+_{[\nu} R^{+\alpha}_{\ \ |\lambda|\beta\kappa]} - 2H^\sigma_{[\nu\beta} R^{+\alpha}_{\ \ |\lambda|\kappa]\sigma} = 0. \tag{6}$$

where $|\lambda|$ denotes no permutations on the index. It further follows from $dH = 0$ that,

$$D^+_\mu H_{\nu\rho\sigma} = \frac{3}{2} R^+_{[\nu\rho\sigma]\mu}. \tag{7}$$

The generalized Ricci tensor is then simply $R^+_{\mu\nu} = R^{+\alpha}_{\ \ \mu\alpha\nu}$ and may be evaluated to be

$$R^+_{\mu\nu} = R_{\mu\nu} + D_\alpha H^\alpha_{\ \nu\mu} - H^{\rho\alpha}_{\ \ \mu} H_{\nu\rho\alpha}. \tag{8}$$

The symmetric part of (8) then provides the usual metric one loop beta function and the antisymmetric part is the corresponding expression for the beta function of the antisymmetric tensor field, both without the usual dilaton part. So the one loop equations of motion for the background after introducing the dilaton part may be expressed as

$$R^+_{\mu\nu} + D_\mu D_\nu \phi + D^\lambda \phi H_{\lambda\mu\nu} = 0. \tag{9}$$

The K-model string backgrounds describing generalization of plane gravitational waves are given as follows [15] with a Brinkmann metric [22],

$$ds^2 = 2dudv + 2A^+_i(u,x)dudx^i + K(u,x)du^2 + dx^i dx_i, \tag{10}$$

and an antisymmetric tensor field;

$$B_{\mu\nu} = \begin{pmatrix} 0 & 1 & A^-_i \\ -1 & 0 & 0 \\ -A^-_i & 0 & 0 \end{pmatrix}. \tag{11}$$

The vector like couplings $A^\pm_i$ plays a significant role in considering these models and provides gauge fields on compactification to lower dimensions. Actualy they are expressed as

$$A^\pm_i = A_i \pm \bar{A}_i \tag{12}$$

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where A_i and $\bar{A}_i$ are the couplings in the corresponding bosonic sigma model [15]. We also consider a dilaton which is taken to be simply a function $\phi = \phi(u)$.

The v isometry of the background is described by the Killing vector l^μ which is given as $(0, 1, 0, \dots 0)$. The only non-zero connections computed from the metric (10) are, Γ^i_{uu}, Γ^v_{uu}, Γ^v_{ui}, Γ^j_{ui} and Γ^v_{ij}. Using these connections, it is easy to show that the null killing vector l^μ is covariantly constant.

The only non-zero independent components of the curvature $R_{\lambda\mu\nu\kappa}$ turn out to be [21] R_{uiuj} and R_{uijk}. These may be evaluated to obtain;

$$R_{uiuj} = \frac{1}{2}\partial_i\partial_j K - \frac{1}{2}\partial_u[\partial_i A_j^+ + \partial_j A_i^+] - \frac{1}{4}\delta^{mn}(F_{jm}^+ F_{in}^+) \tag{13}$$

and

$$R_{uijk} = \frac{1}{2}\partial_i[F_{kj}^+] \tag{14}$$

where

$$F_{jk}^\pm = (\partial_j A_k^\pm - \partial_k A_j^\pm). \tag{15}$$

The field strength of the antisymmetric tensor $B_{\mu\nu}$ in the standard form is $H = \frac{1}{2}dB$ and this upon evaluation gives the only non zero independent component as,

$$H_{uij} = -\frac{1}{2}F_{ij}^-. \tag{16}$$

It is now a straightforward exercise to cast this background in the non-symmetric connection framework described earlier. We essentialy describe here the approach of Duval et. al. [17] and also include the dilaton. Notice first, that the set of the non zero connections remains the same as for the symmetric case for the backgrounds under consideration cf eqns.(10,11). The only connections which are modified by the torsion contribution are Γ^v_{ij} and Γ^j_{ui}. The non-zero components of the generalized curvature are then

$$R^{+i}_{jku} = -\frac{1}{2}\delta^{il}\partial_k[\partial_l A_j^+ - \partial_j A_l^+ - \partial_l A_j^- + \partial_j A_l^-] \tag{17}$$

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$$R^{+i}{}_{ujk} = \frac{1}{2}\delta^{il}\partial_l[\partial_k A_j^+ - \partial_j A_k^+ - \partial_k A_j^- + \partial_j A_k^-] \tag{18}$$

and

$$\begin{aligned}
R^{+i}{}_{uju} &= \delta^{il}[\partial_j(\partial_u A_l^+ + \frac{1}{2}\partial_l K) + \frac{1}{2}\partial_u(\partial_l A_j^+ - \partial_j A_l^+ - \partial_j A_l^- + \partial_l A_j^-)] \\
&+ \frac{1}{4}[(F_{jk}^+ + F_{jk}^-)(F_k^{+i} - F_i^{-k})]
\end{aligned} \tag{19}$$

The tree level equations of motion are then obtained from the generalized Ricci tensors and the dilaton contributions. Using eqns. (10, 17, 18, 20) we obtain;

$$R_{ju}^+ = -\frac{1}{2}\partial^i[\partial^i A_j^+ - \partial_j A_i^+ - \partial_i A_j^- + \partial_j A_i^-] = 0 \tag{20}$$

$$R_{uj}^+ = \frac{1}{2}\partial^i[\partial^i A_j^+ - \partial_j A_i^+ + \partial_i A_j^- - \partial_j A_i^-] = 0 \tag{21}$$

and

$$\begin{aligned}
R_{uu}^+ + 2\partial_u^2\phi &= \partial^i(\partial_u A_i^+ + \frac{1}{2}\partial_i K) - \frac{1}{4}(\partial^k A_i^+ - \partial_i A_k^+)^2 \\
&+ \frac{1}{4}(\partial^k A_i^- - \partial_i A_k^-)^2 \\
&= 0,
\end{aligned} \tag{22}$$

where $2\partial_u^2\phi$ is the dilaton contribution. Notice that the dilaton term coupling to the torsion in the beta function equation, $D^\lambda\phi H_{\lambda\mu\nu} = 0$ as a consequence of the Killing equations.

The arguments for the higher order corrections to these equations of motion (20,21, 23) as presented in [17], proceeds as follows. These higher order corrections are in fact all possible non-zero rank two tensors obtained from appropriate contractions of the generalized curvature, torsion, dilaton and their covariant derivatives (the covariant derivative of H may be expressed in terms of R^+ from eqn (6). This is of course because the eqns. of motion are field equations for rank two tensors. If the vector like couplings satisfy the condition

$$A_i^+ = \pm A_i^+ \tag{23}$$

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then it is obvious from eqn. (17, 18) that R^+_{uijk} vanishes. This translates to the condition, $G_{ui} = \pm B_{ui}$ for the metric and the antisymmetric tensor components. From the bosonic sigma model point of view this essentialy means that the couplings $A_i = 0$ and $\bar{A}_i = 0$ from eqn. (12) i.e. the vector like couplings in the sigma model are chiral. With this condition (for eg. we will consider the positive sign) the conformal invariance conditions at one loop reduce to only two equations namely;

$$R^+_{uj} = \partial^i(\partial_i A^+_j - \partial_j A^+_i) = 0 \tag{24}$$

and

$$R^+_{uu} + \partial^2_u \phi = \partial^i(\partial_u A^+_i - \frac{1}{2}\partial_i K) + \partial^2_u \phi = 0. \tag{25}$$

In consequence of the vanishing of R^+_{uijk}, the generalized curvature with three spatial indices, all terms involving R^+ and H, of rank two are identicaly zero. This is because they require contraction of at least one u index which is not allowed by the specific form of the metric. Next in the terms with R^+ and its derivatives, notice that D^+_v operation on the background is zero valued, as both $\Gamma^+_{v\cdot}$ and $H_{v\cdot\cdot}$ (where the ellipses denote other indices) are zero owing to the isometry of the background in v. In consequence the operation D^{+u} is also zero from the form of the metric. Hence all such terms involving these derivatives are identicaly zero as they require contractions of the index u. Simmilar considerations show that terms involving D^+R^+ and D^+H^+ are also vanishing.

Finally we consider terms in single R^+ of the form $D^{+\mu}D^{+\rho}R^+_{\mu\nu\rho\sigma}$. This may be expressed in terms of the derivatives of the generalized Ricci tensor using the generalized Bianchi identity. It is possible to show using this [17] that all such terms vanish. Similarly using the Killing equations for the other backgrounds it may be shown that all higher order terms involving the dilaton and its derivatives are also identicaly zero [16, 17, 20, 21]. It turns out, at least at two loop level that all possible non-zero higher order contributions may be reduced to the form [15]

$$\beta_{\mu\nu} = Y^{+\lambda\rho\sigma}{}_\nu R^+_{\mu\sigma\lambda\rho} \tag{26}$$

where $Y^{+\lambda\rho\sigma}{}_\nu$ is a function of R^+ and derivatives of H. If appropriate field redefinitions exist such that this is continued at higher orders and R^+_{uijk} is zero from eqn. (23) then all higher order terms vanish. This is because at least one of the indices to be contracted in (26) must be u and it has been shown that all such contractions are identicaly zero. So we may conclude that the K-model string backgrounds are exact subject to these conditions. This conclusion has been corroborated by an explicit sigma model based proof [15]. This cancellation is of course a result of the covariantly constant null Killing isometry which these backgrounds admit and the condition (23). This concludes the description of K-model string background in the non-symmetric connection framework [17]. In the next section we describe the technique of generating non trivial R-R sector through the S-duality transformations and cast the type IIb backgrounds obtained in the non-symetric framework prior to examining the issue of all higher order corrections to the equations of motion.

3. Type IIB K-model Superstring Backgrounds.

The complete massless bosonic field content of type IIb superstring backgrounds consists of the string frame metric $G_{\mu\nu}$, two 2-form gauge fields $B^{(A)}_{\mu\nu}$ with $A = (1, 2)$, two scalars ϕ and χ from the NS-NS and the R-R sectors respectively and a real self-dual 4-form $D_{\mu\nu\rho\sigma}$. As mentioned earlier the type IIB superstrings in D=10 have a global $SL(2, R)$ symmetry of the equations of motion of the effective supergravity theory [5]. The action of the $SL(2, R)$ transformations on the background fields may be described as follows. We define the complex scalar field $\lambda = \chi + ie^{\phi}$. Then if Λ is an $SL(2, R)$ matrix such that

$$\Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix}, \tag{27}$$

with $ad - bc = 1$, the action on the type IIb background fields are

$$G'_{\mu\nu} = \mid c\lambda + d \mid G_{\mu\nu}, \tag{28}$$

$$\lambda' = \frac{a\lambda + b}{c\lambda + d}, \tag{29}$$

and

$$H'^{(A)}_{\lambda\mu\nu} = \Lambda H^{(A)}_{\lambda\mu\nu}, \tag{30}$$

where $H^{(A)}$ are the field strengths corresponding to the two 2-forms $B^{(A)}$, (A=1, 2), from the NS-NS and the R-R sectors respectively.

We may consider the K-model backgrounds described in the last section to be a special case of the type IIB with the R-R fields $B^{(2)} = 0$, $\chi = 0$ and $D = 0$. So we have $\lambda = ie^{\phi}$ as $\chi = 0$ and implementing the $SL(2, R)$ transformations (28, 29, 30) on this background we obtain the complete type IIB background with the following field content;

$$G'_{\mu\nu}(u, x) = f(u)G_{\mu\nu}(u, x), \tag{31}$$

where $f(u) = [d^2 + c^2 e^{-2\phi(u)}]^{\frac{1}{2}}$ and

$$\chi'(u) = \frac{1}{f(u)^2}[db + ac\ e^{-2\phi}], \tag{32}$$

$$\phi'(u) = \phi(u) + 2\ ln\ f(u). \tag{33}$$

For the 3-form field strength $H^{(A)}$, $(A = 1, 2)$ we have

$$H'^{(1)}_{\lambda\mu\nu} = dH^{(1)}_{\lambda\mu\nu}, \tag{34}$$

and

$$H'^{(2)}_{\lambda\mu\nu} = bH^{(1)}_{\lambda\mu\nu}. \tag{35}$$

After a rescaling $f(u)du = dU$ and rewriting U as u leads to the general form;

$$ds^2 = 2dudv + 2f(u)dx^i dx_i + 2A^+_i(u, x)dudx^i + K(u, x)du^2. \tag{36}$$

In subsequent discussions we drop the primes on the type IIB background fields generated by the $SL(2, R)$ transformations from the K-model backgrounds and also employ the same notation for the transformed functions $A^{\pm}_i$ and K.

Notice that the null Killing isometry has been preserved by the S-duality. Using the expressions for the metric from eqn. (36) we observe that the only non-zero components of the Christoffel connections are, Γ^v_{uu}, Γ^i_{uu}, Γ^v_{ui}, Γ^j_{ui} and Γ^v_{ij}. From the non-zero connections it follows that the null Killing isometry is still covariantly constant which is expected as the S-duality respects space-time geometries. The only non-zero independent components of the curvature tensor [21] turn out to be as follows,

$$
\begin{aligned}
R_{uiuj} &= \frac{1}{2}\partial_i\partial_j K - \frac{1}{2}\partial_u[\partial_i A_j^+ + \partial_j A_i^+] \\
&+ \frac{1}{2}\partial_u^2 f \delta_{ij} - \frac{1}{4f^2}[\delta_{ij}(\partial_u f)^2 + F_{jm}^+ F_i^{+m}]
\end{aligned}
\tag{37}
$$

and

$$
R_{uijk} = \frac{1}{2}\partial_i[\partial_k A_j^+ - \partial_j A_k^+].
\tag{38}
$$

Notice that expression for R_{uijk} is unchanged from that for the purely NS-NS background. Having obtained the complete type IIB background with non-zero R-R field content, in the next Section we proceed to cast these into the non-symmetric connection framework described earlier.

4. Type IIB Background and the Non-Symmetric Connection Approach.

In this section we present the non-symmetric connection approach for the type IIB backgrounds obtained by us [21] with non-zero R-R field content. The definition of the non-symmetric connection in eqn. (2) requires to be modified due to the the two torsions arising from the NS-NS and the R-R sectors. As the torsions are additive the simplest modification which suggests itself is,

$$
\mathcal{H}_{\lambda\mu\nu} = H^{(1)}_{\lambda\mu\nu} + H^{(2)}_{\lambda\mu\nu}
\tag{39}
$$

$H^1 = dH$ and $H^2 = bH$ where d and b are just the elements of the $SL(2, R)$ matrix Λ. Notice that such a definition is consistent with general covariance and space-time tensor gauge invariance. This also preserves all the relations for the non-symmetric

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connection presented in Section-2 including the generalized Bianchi identities with $\mathcal{H}$ replacing H in (2). In fact one may use a linear combination of the two torsions also where the coefficients would be restricted by the generalized Bianchi identity. Notice that for the backgrounds being analysed, the definition (39) corresponds to a simple constant scaling by the quantity $(d + b)$ of the starting NS-NS 2-form and consequently implies a simmilar scaling on the antisymmetric tensor components.

We now proceed to explicitly compute the generalized curvatures and Ricci tensors for the background described in the last section. As earlier the set of non-zero connections remain the same as in the symmetric case and the modifications to their forms identical. The only non-zero independent components of the generalized curvature are

$$R^{+i}_{jku} = -\frac{1}{2f(u)}\delta^{il}\partial_k[\partial_l A^+_j - \partial_j A^+_l - \partial_l A^-_j + \partial_j A^-_l] \tag{40}$$

$$R^{+i}_{ujk} = \frac{1}{2f(u)}\delta^{il}\partial_l[\partial_k A^+_j - \partial_j A^+_k + \partial_k A^-_j - \partial_j A^-_k] \tag{41}$$

and

$$
\begin{aligned}
R^{+i}_{uju} &= \frac{\delta^{il}}{2f}[\partial_l\partial_j K - \partial_u(\partial_l A^+_j + \partial_j A^+_l) + \partial_u(\partial_l A^-_j - \partial_j A^-_l) \\
&+ \partial^2_u f \delta_{jl} - \frac{1}{2}\delta_{jl}(\partial_u f)^2 \\
&+ \frac{1}{2f}(F^{+n}_j + F^{-n}_j)(F^+_{ln} - F^-_{ln})]
\end{aligned}
\tag{42}
$$

Where now the A^-_j have been suitably scaled to accomodate for the combination defined in eqn. (39). The tree level supergravity equations of motion for the metric and the antisymmetric tensor may now be once again expressed in terms of the generalized Ricci tensors combined with the appropriate scalar field contributions. The equations of motion for the scalar fields must be added on by hand separately. The torsion equations would be identical in form to eqns. (20, 21), except that the A^-_j have now been suitably scaled due to relation (39). The only change will involve the

metric equation where additional contribution from the R-R scalar must be included. The (uu) component of the Ricci tensor is just

$$R^+_{uu} = \frac{1}{f^2}[\frac{1}{2}\partial^i\partial_i K + 4\partial_u^2 f - 2(\partial_u f)^2 + \frac{1}{4}(F^+_{nl} - F^-_{nl})^2] \tag{43}$$

These along with appropriate scalar contributions essentialy describes the tree level type IIB supergravity equations of motion for the backgrounds under consideration.

Notice that the tree level equations of type IIB supergravity in the string frame metric in ref. [6] reduces for the backgrounds under consideration to just four independent equations as $F_5 = 0$, the ones for the torsion being identical. These are the equations for the metric, the two scalars and the antisymmetric tensor In particular we have the equation for the metric as

$$R_{uu} = 4\partial_u\phi\partial_u\phi - \frac{e^{2\phi}}{2}\partial_u\chi\partial_u\chi + w(u)H^2_{uu} = 0 \tag{44}$$

where $w(u)$ is a complex scalar function of u. It is possible to obtain (44) from (43) with a suitable scaling of the metric and addition of appropriate scalar contributions. Notice that the contribution $D^\rho H_{\rho\mu\nu}$ to the equation is zero owing to the torsion equations of motion. For ϕ χ constant we have from ref. [6] that the function $w(u)$ is a constant and with appropriate scaling we get back eqn. (43). The other equations from ref. [6] for the background under consideration are;

$$D^2\phi = 0. \tag{45}$$

$$D^2\chi = 0. \tag{46}$$

$$D^\mu H_{\mu\nu\rho} = 0. \tag{47}$$

and

$$F_5(D) = \tilde{F}_5(D) = 0 \tag{48}$$

Having presented the tree level equations of motion for the effective type IIB supergravity theory we now proceed to address the question of the higher order

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α'contributions to these equations. All the arguments presented in Section-2 for the higher order corrections to the purely NS-NS case are also valid for the type IIB case. This is owing to the fact that the 2-forms in the NS-NS and the R-R sectors turn out to be just simple scalar multiples of the starting NS-NS 2-form and also because the generalized curvature has the same non zero components. Using the Killing equations it may be shown that all possible scalar and rank five antisymmetric tensor corrections, if any to the trivialy self dual condition for F_5 are also zero [21]. If we now adopt the condition for the chirality of the vector like couplings, namely

$$A_i^+ = \pm A_i^-, \tag{49}$$

or equivalently $G_{ui} = B_{ui}$ we have the curvature component R^+_{uijk} with three transverse indices identicaly zero. Using the arguments outlined in Section-2 for the purely NS-NS backgrounds it may be shown that the type IIB backgrounds under consideration are also exact to all orders in α'. Obviously this is subject to the fact that all higher order terms as earlier, may be expressed in the form $Y^{+\lambda\rho\sigma}_\nu R^+_{\mu\sigma\lambda\rho}$ through field redefinitions.

The argument is not completely rigourous in the absence of an explicit sigma model description for the non-p-brane like type IIB superstring backgrounds which are considered here. However, a stronger argument for the existence of the field redfinitions may be obtained by explicitly checking the generic non-zero higher order correction terms that may arise. Notice that the only higher order terms that may occur are in the equation following from the (uu) component of the Ricci tensor owing to the form of the background metric. Following arguments simmilar to those in Section-2 it may be shown that all higher order corrections to the other equations of motion vanish identicaly.

The only non-zero higher order correction at two-loop order to eqn. (44) that may arise are from $R_{uijk}R_u^{ijk}$ and $D_k H_{uij}^{(A)} D^k H_u^{(B)\ ij}$ where $(A, B = 1, 2)$ refers to the NS-NS and the R-R sector and R is the usual curvature. This is obvious from the

form of the background as all other higher order terms vanish as a consequence of the covariantly constant null Killing isometry. Assuming that the coefficients are just numerical constants we have the explicit higher order term;

$$\beta_{uu}^2 = pR_{uijk}R_u^{\;ijk} + q_{AB}D_kH_{uij}^{(A)}D^kH_u^{(B)\;ij} \tag{50}$$

where p and q_{AB} are constants. This reduces for the background in our case to

$$\beta_{uu}^2 = pR_{uijk}R_u^{\;ijk} + qD_kH_{uij}D^kH_u^{\;ij} \tag{51}$$

where H is the starting NS-NS torsion and q is another constant. The terms in eqn. (51) are the only non-zero terms in the contraction, $R_{uijk}^+R_u^{+ijk}$. This strongly suggests that under suitable redefinitions eqn. (51) reduces to $R_{uijk}^+R_u^{+ijk}$. However as $R_{uijk}^+ = 0$ with the condition defined by eqn. (23) the contraction of the two generalized curvatures vanish. This shows that all higher order contributions at two-loop orders to the equations of motion are identicaly zero subject to appropriate field redefinitions. It is quite plausible that this generalizes to arbitrary higher orders for the background under consideration. This is because, firstly the corrections appear only for the uu component of the metric equation and all higher order terms must arise from terms of the schematic form $D...R_{uijk}D...R_{uijk}$ and $D...H_{uij}D...H_{uij}$ suitably contracted. These may always be combined under suitable field redefinition to give terms of the generic form as defined earlier. The results indicate that the type IIB S-duality transformations preserve the α' structure of the backgrounds considered here. Whether this assertion is true in general is an interesting issue to explore.

5. Summary and Discussions.

To summarize, we have examined the issue of higher order corrections to the equations of motion of type IIB superstring backgrounds obtained by us recently [21]. These ten dimensional backgrounds were obtained through the S-duality transformations of plane wave K-model backgrounds with purely NS-NS field content. They describe the most general class of type IIB superstring backgrounds with a covariantly constant

null Killing isometry and a flat transverse part in ten dimensions involving non-zero Ramond-Ramond field content. A geometrical approach in the non-symmetric connection framework was adopted for this exercise as a sigma model interpretation for these backgrounds are still obscure. The non-symmetric connection approach was generalized to these type IIB superstring backgrounds by using a additive combination of the two torsions from the NS-NS and the R-R sectors. From a geometrical analysis of the tensor structure of the possible higher order terms we show that the type IIB K-model superstring backgrounds are exact to all orders in α'. This is subject to the condition that the vector like K-model couplings enter the theory in a chiral fashion and the existence of certain field redefinitions. We obtain a strong argument in favour of the existence of the field redefinitions through an explicit representation of the generic higher order corrections at two-loops. With the chirality condition the backgrounds obtained by us are the type IIB analogs of chiral plane waves in ref. [15].

Our results indicate that the $SL(2, R)$ S-duality of type IIB in ten dimensions preserves the structure of α'corrections of the chiral plane wave backgrounds. Of course this is a direct consequence of the covariantly constant null Killing isometry and interelation between the metric and the antisymmetric-tensor for the special backgrounds that we have considered. Whether such an assertion is true in general seems to be an interesting issue to investigate. A general proof seems quite non-trivial especialy as the structures of higher order corrections are not known explicitly. These are quite difficult to obtain even for p-brane backgrounds for which an explicit Dirichlet sigma model description is available [9]. Furthermore a sigma model interpretation for the non-brane like backgrounds with R-R fields discussed here is still an obscure issue which needs to be addressed. Our results strongly indicate that such an exact conformal field theory description should exist. Some of these type IIB backgrounds with non-trivial R-R fields have been recently identified with Toda models in the context of their applications to string cosmology [25]

It has been shown [12] that the NS-NS chiral plane wave backgrounds may be

embedded both in heterotic and type I superstrings and for the latter case preserves only half of the space-time supersymmetry in ten dimensions. The status of unbroken space-time supersymmetry would be also interesting to investigate in the full type IIB case discussed here. The results will have implications for these type IIB backgrounds to be solutions in presence of local string loop corrections as well. These issues are currently under consideration.

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